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Modelling eruption cycles and decay of mud volcanoes

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ABSTRACT

Recent debates about the eruptive behavior of mud volcanoes and their activation mechanisms have been driven particularly by the LUSI eruption in Indonesia that resulted in huge commercial and cultural damages. Numerical modeling of mud volcanoes, of which few exist, can provide insight into eruptive behavior and contribute to hazard evaluation. In this paper, we present a simple model to describe fluid escape from an underground reservoir through a conduit, extruded as a mud volcano at the surface. The governing equations result in oscillatory behavior, and we study the influence of changes in rheological properties of surrounding rock and fluid characteristics of the mud on extrusion dynamics. We focus on understanding long-term eruption behavior, flow cycles, and decay factors. Model results can be used to estimate the discharge rates and extruded volume from assumptions on the mud reservoir and conduit, or conversely, the reservoir or conduit properties from discharge rates.

1. Introduction

Mud volcanoes result from the extrusion of gas- and water-saturated mud both in sub-aerial and in sub-marine environments. This semi-liquid is forced through openings in the upper crust, sometimes producing massive quantities of mud on the surface, as evidenced by the recent LUSI mud volcano in Java, Indonesia. Globally, the distribution of mud volcanoes shows about 1800 individual sites (Dimitrov, 2002), and extraterrestrial occurrences are also documented (Fortes and Grindrod, 2006; Skinner and Tanaka, 2007; Skinner and Mazzini, 2009). The basic mechanism of mud volcano formation is the release of high-pressure mud trapped at depth. Triggering mechanisms of the volcanoes are still debated, but various hypotheses include earthquake-triggering, fault failure, and drilling (Manga et al., 2009, and references therein).

The morphology of mud volcanoes varies, and includes conical vents and bubbling mud pools (Fig. 1(a)). Some mud sources are shallow <1 km, while others are fed by reservoirs at depths of up to 6 km. Vents range from the centimeter scale to several hundreds of meters (Aslan et al., 2001; Mazzini et al., 2009a,b). Extruded material can include mud, gas, boulders of clay or other solid material, indicating that dike-like conduits form in response to over-pressured fluids flowing along permeable fractures, eroding the wall rock and evolving to an open vent (Bonini, 2007).

This study investigates some mechanical considerations of mud volcanism. While some mud fields are continuously active (with ongoing substantial seepage for more than 60 years of observation), other areas exhibit an alternation between periods of eruption and relative quiescence. The time intervals between significant material escape during dormant stages vary from minutes to several days, but in many cases show a cyclic behavior. For example, the Dashgil mud volcano in Azerbaijan is characterized by continuous pulsating venting of mud, water and gas (Hovland et al., 1997; Mazzini et al., 2009a,b). Onshore in Trinidad, it has been shown that the vertical conduits allow the escape of gas-charged methane-rich cold seeps. The mud volcanoes have cyclic phases of eruptions, where the initial sedimentary mobilization could have occurred from pore water in deep sandy reservoirs (Deville et al., 2006). For the mud field along the Pede-Apennine margin, fault failure cycles (tectonic loading and unloading) are hypothesized to promote a long-term fluid release cycling, during which over-pressured fluids are periodically discharged from a reservoir through the creation or the reactivation of fractured systems (Bonini, 2007). A highly studied case is the eruption of mud and gas called LUSI, that started 29th of May 2006 in North east Java (Davies et al., 2008; Mazzini et al., 2007). The discharge rate rose from 5000 to 120,000 m^3/d during the first eleven weeks, flooding large area of the Sidoarjo village. The mud flow then was observed to pulsate, and the extruded volume again increased dramatically following earthquake swarms. Although LUSI was perturbed several times, the mud flow shows a clear tendency to pulsate every few hours and to erupt in changing cycles – an important characteristic, that has lasted for more than two years and continues still.

The observed oscillatory behavior of mud volcanoes is the focus of the conceptual and mathematical model proposed here. We

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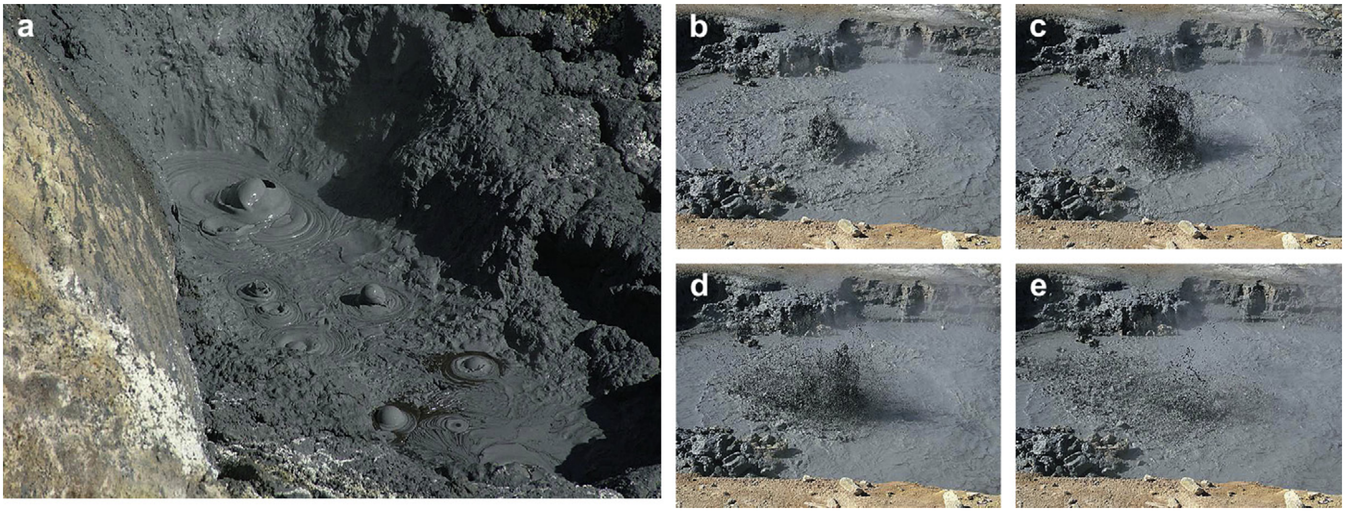

Fig. 1. (a) Mud volcanoes belonging to a mud pool near Krafla volcano, Iceland. (b)–(e) Detailed view on an eruption. Namaskard region, Iceland.

focus on the periodic characteristics and develop mathematical model equations where the solution for the material discharge rate oscillates naturally. Results are presented that describe the solution and its dependence on initial conditions and physical parameters such as fluid properties and characteristics of the mud volcano. In the case where an eruption or seepage process decays with time, results show the evolution towards an oscillatory state about equilibrium. Integration of the calculated discharge rate provides information about the volume extruded at the surface, allowing comparisons and observational constraints on processes occurring at depth and estimates on societal hazards.

2. Conceptual model

Fig. 2 shows the conceptual model. We assume that a mud volcano consists of two components; a mud reservoir with volume V at a certain depth beneath the surface, and a vertical cylindrical conduit with cross-sectional area A and height h.

The choice of a two-component system is not arbitrary, but arises from the observation that mud eruptions are activated and influenced by pressure or rheological changes at depth (Bonini, 2007; Manga et al., 2009; Mazzini et al., 2009a,b). As a first approach, we investigate a scenario where the reservoir is filled with mud of pressure p and density ρ, related through an adiabatic condition for mud compressibility β. Material enters the reservoir across its surface S with a volumetric influx rate I, producing a pressure increase inside the reservoir. In addition, we allow a possible secondary pressurisation of the system due to a reservoir deflation. The overpressure forces the mud to leave the system with a volumetric discharge rate Q via mud ascent inside an open cylindrical conduit. More realistic scenarios should encompass different types of observed mud volcano conduit styles, for example, conical vents (Deville et al., 2006; Dimitrov, 2002), fracture zones filled with permeable rock (Ingebritsen and Rojstaczer, 1993) or the combination of fractured systems ending up in open conduits near the surface.

This model set-up contains many parameters that influence the behavior. Instead of a detailed study of all possible parameter combinations, we choose to make some simplifications to reduce the mathematical complexity. We investigate the evolution of the discharge rate function of a mud volcano in time, after the fluid escape has already been initiated by an unidentified triggering mechanism (Mazzini et al., 2009a,b), and after a conduit evolved from an initial venting process (Gisler, 2009). Although this simple model ignores potentially important details of an eruption, this formulation results in essential characteristics of the mud volcano system, most importantly the long-term oscillatory behavior.

The primary constraints are the geometry of the reservoir and conduit. We assume a cylindrical conduit, and ignore variations in cross-sectional area due to erosion of the conduit wall. For the reservoir, we adopt models developed for magma chambers (Denlinger and Hoblitt, 1999; Iverson et al., 2006; Iverson, 2008). For instance, the fluid inside the reservoir balances pressure changes via density changes, the geometry is assumed spherical or cubic, the influx velocity of additional material is constant, and the reservoir volume change is either zero (non-deflating case) or negative (deflating reservoir).

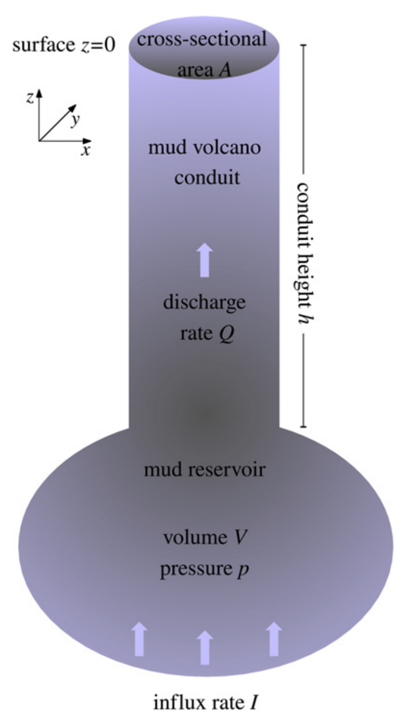

Fig. 2. Schematic illustrating the basic concept of our two-component mud volcano model (not to scale).

Second, we make simplifications on the mud properties. In general, mud most likely behaves as a compressible non-Newtonian fluid, requiring the use of density and viscosity functions (Mazzini et al., 2009a,b). Compressibility effects are important when compaction exceeds percolation effects (Gisler, 2009). Therefore, we regard mud compressibility inside the pressurised reservoir, but inside the conduit we adopt an isochoric flow condition for the mud discharge rate. Thus, velocity and density changes with position are negligible, implying zero density changes with time. This simplification generally may not be applied when analysing velocity profiles or when determining resonance effects due to the flow of fluids with Mach numbers over 0.3 (the dimensionless ratio of flow velocity to speed of sound in the material), but regarding global solutions of compressible flow problems this adoption is feasible (Desjardins and Grenier, 1999; Lions and Masmoudi, 1998). Therefore density changes may be neglected and thus mud density ρ is taken constant in our conduit model. We also assume a constant dynamic viscosity because, after the onset of eruption, the temperature gradients are low between the source and the surface. (In the case of LUSI, the exit mud temperature was about 100 °C, so is close to the temperature at the source (Mazzini et al., 2009a,b)). More advanced models should include a temperature-dependent viscosity, but this is beyond the scope of the present study. Under certain conditions, mud can behave thixotropically, but this property is more relevant for the triggering process at depth than for influencing the ongoing discharge. For discussion on the influence of non-constant viscosity values on the discharge rate behavior, we refer to Melnik and Sparks (1999, 2005); Wylie et al. (1999) and Yoshino et al. (2007).

3. Mathematical formulation

The governing equations for the model are conservation of mass and momentum for the mud inside reservoir and conduit. The constitutive model is described below. Temperature is assumed to play a minor role in mud volcanoes, and we introduce an adiabatic condition for mud compressibility β. Within the vertical system leading to the surface, we treat mud ascent as an isochoric flow of a Newtonian fluid with constant density ρ and dynamic viscosity μ. We ignore variations of mud properties and conduit geometry with depth. Discharge velocity u is driven by reservoir pressure, and is resisted by the mud weight and a drag force depending on conduit characteristics. We present two constraints on the reservoir, where differences in its behavior imply different influx functions I, and combine these reservoir models with the conduit set-up to deduce equations describing a wide range of mud volcano eruption systems. All symbols with units are listed in Table 1.

3.1. Governing equations

For ascending mud during a mud volcano eruption with density ρ inside a cylindrical conduit with constant cross-sectional area A, conservation of mass in the one-dimensional case is:

$$\frac{d\rho}{dt}+\frac{d\rho u}{dz} = 0, \tag{1}$$

and the conservation of momentum is described by the 1D Navier–Stokes equation with negligible convection:

$$\frac{d\rho}{dt}u+\frac{du}{dt}\rho = -\frac{dp}{dz}-\rho g-\frac{8\pi\mu u}{A}, \tag{2}$$

where u denotes the mud ascent velocity, p is the mud pressure, g is gravitational acceleration, z is the vertical direction (positive upwards), t is the time and the last term denotes a drag resistance that depends on conduit characteristics. Here, for an open vertical system, this drag force is inferred from Poiseuille flow assumptions with constant dynamic viscosity μ and conduit cross-sectional area A, geared to models derived for volcanic settings (Barmin et al., 2002; Costa et al., 2007; Dobran, 2001).

Table 1
Parameter symbols, description and their values or conversion used for numerical computation.

Symbol	Value	Unit	Definition
A	$0.1–10^3$	m^2	conduit/column cross-sectional area
β	$10^{-2}–10^{-8}$	Pa^{-1}	mud compressibility inside reservoir
c	$c<0$	none	dimensionless number for influx rate solution
g	9.81	$m\ s^{-2}$	gravitational acceleration
h	500–4000	m	conduit/column height
I_0	0.1–2	$m^3\ s^{-1}$	initial mud influx rate
I_{const}	0.1–2	$m^3\ s^{-1}$	constant mud influx rate
μ	$10^{-6}–10^2$	Pa s	mud viscosity
Q_0	0–2	$m^3\ s^{-1}$	initial mud discharge rate
r_0	$\propto \sqrt[3]{V_0}$	m	initial reservoir radius
ρ	500–2000	$kg\ m^{-3}$	mud density in conduit
S_0	$\propto \sqrt[3]{V_0^2}$	m^2	initial reservoir surface
u_0	Q_0/A	$m\ s^{-1}$	initial mud velocity
v_0	I_0/S	$m\ s^{-1}$	initial mud influx velocity
V_0	$10^6–10^{12}$	m^3	initial reservoir volume

Rearranging equations (1) and (2), replacing the pressure gradient by a linear pressure increase with depth h, and integrating over the constant cross-sectional area A, we formulate an ascent equation as a function of discharge rate Q:

$$\frac{dQ}{dt} = \frac{A}{\rho h}p-Ag-\frac{8\pi\mu}{\rho A}Q \tag{3}$$

for mud flow inside an open conduit. The advantage of this equation is that estimates of the extruded mud volume at the surface can be obtained by time integration.

We assume the following boundary conditions for equation (3):

$$z = 0: \quad p = 0, \tag{4}$$

$$z = -h: \quad \frac{dp}{dt} = \frac{1}{\beta V}\left(I-Q-\frac{dV}{dt}\right), \tag{5}$$

where V is the reservoir volume, I is the volumetric mud influx rate into the reservoir, Q is the volumetric mud discharge rate into the conduit and h is the conduit height (being equal to the reservoir depth). Equation (5) is derived from the equation for conservation of fluid mass inside the reservoir

$$\rho\frac{dV}{dt}+V\frac{d\rho}{dt} = \rho(I-Q), \tag{6}$$

combined with the mud compressibility

$$\beta = \frac{1}{\rho}\frac{d\rho}{dp} \tag{7}$$

that describes the density response due to pressure changes.

For $dV/dt<0$ in the last term of equation (5), the boundary condition provides a reservoir deflation process, whereas for the case of a non-deflating reservoir the volume V is constant, thus the last term of equation (5) is zero, and the boundary condition at $z=-h$ reduces to

$$\frac{dp}{dt} = \frac{1}{\beta V}(I-Q). \tag{8}$$

This formulation shows that, inside the reservoir pressure decreases due to the mud discharge rate Q, and increases due to the mud influx rate I or alternatively by the reservoir deflation dV/dt.

3.2. Supplementary equations

We make two assumptions regarding the conditions inside the mud reservoir beneath the vertical conduit. First, we assume a constant reservoir volume V, and second we allow the reservoir to deflate linearly with its radius r. These are strong assumptions, but they show how the model is affected by constant and decreasing fluid sources. A more likely case is a combination of the two, and can be constrained by the mud volcano of interest. For example, in cases of observed subsidence (such as the LUSI eruption) it is reasonable to assume a deflating reservoir (Mazzini et al., 2007).

For a steady-state condition of the reservoir volume V, we assume a constant material influx at velocity v = constant and $dV/dt = 0$. Since the influx rate I depends on the influx velocity v times the reservoir surface S, the influx rate I is constant, and we denote it by I_{const}.

For a deflating mud-filled reservoir we use the elementary relations between the reservoir surface S and reservoir volume V (for instance, a spherical or cubic reservoir geometry yield the dependencies $dV = Sdr$ and $S \propto \sqrt[3]{V^2}$, to within a numeric factor), and we assume a constant, negative radius decrease with time $dr/dt < 0$, resulting in the volume change function

$$\frac{dV}{dt} = S\frac{dr}{dt}, \tag{9}$$

with $|S(dr/dt)| \leq Q$, since we restrict the reservoir deflation to a smaller rate than the discharge rate. Retaining a constant material influx velocity v = constant, and assuming again the continuity equation $I = vS$ for the volumetric flow rate I of mud through the reservoir surface S, the decreasing reservoir surface has a direct impact on the influx rate. The change rate of I is related to the change rate of S via $SdI/dt = IdS/dt$, which can be used to substitute I or S where necessary. For instance, we obtain

$$\frac{dI}{dt} = c\frac{I^2}{V}, \tag{10}$$

where the negative constant $c < 0$ contains dr/dt, v and fundamental spherical (or cubic) geometry parameters from the relation between surface, volume and radius of the reservoir. We point out that other influx functions are possible, since many approaches already exist to the problem of fluid influx into a chamber, e.g. allowing periodic supply rates by making stronger assumptions on influx velocity or rate (Dobran, 2001; Georgiou, 1996). We proceed using equation (10), to avoid arbitrary and non-physical assumptions on the influx rate, and to keep the model transparent.

3.3. Results

In order to make estimates on the extruded volume at the mud volcano surface, we reformulated the fundamental model equations

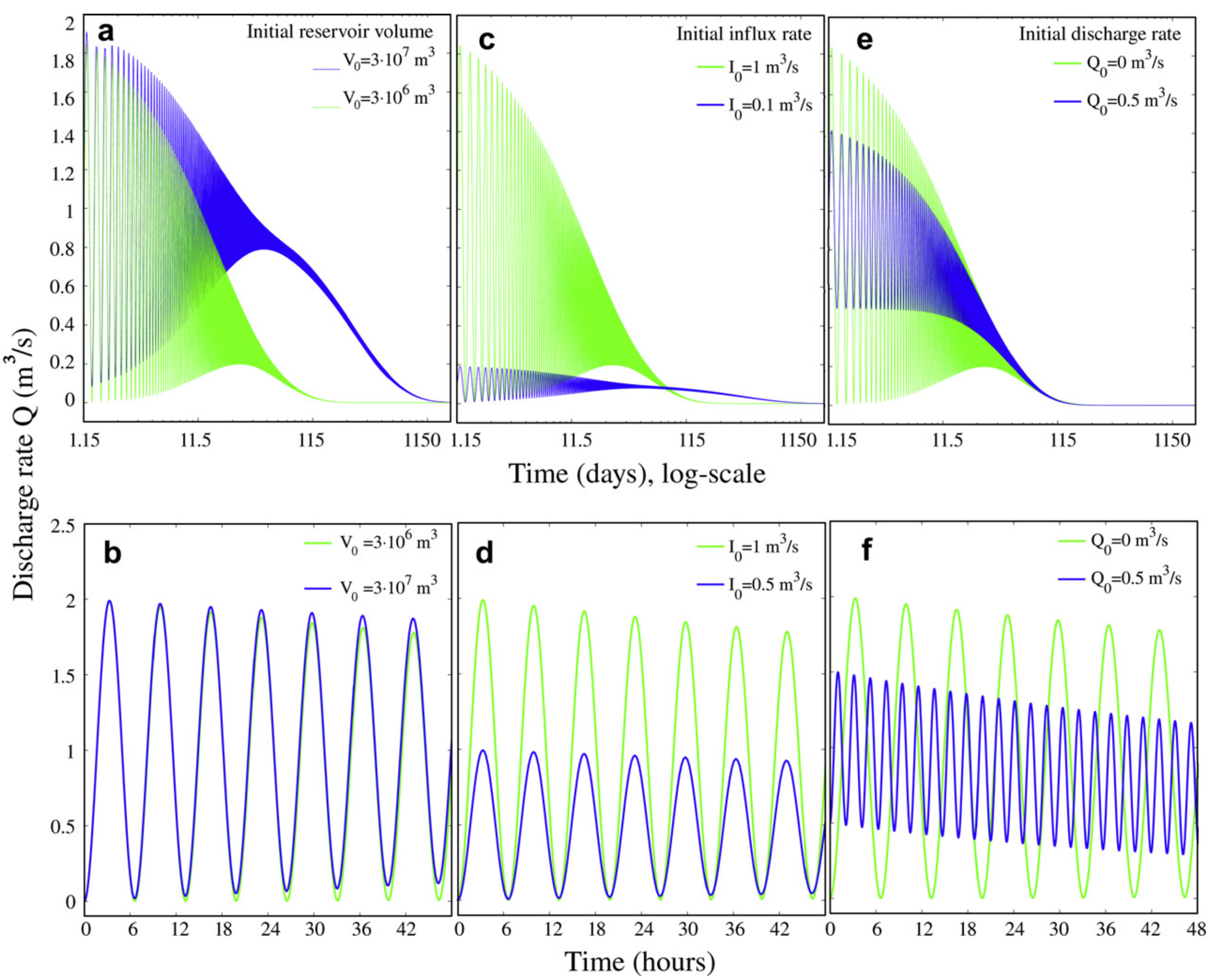

Fig. 3. Discharge rate Q solutions for a change in initial conditions. (a) Decreasing V_0, a faster decay can be seen. (b) Close-up, no period changes. (c) A decrease in I_0 changes the extrusion intensity and decay. (d) Detailed view shows amplitude changes by factor 0.5, while period remains the same. (e) Long-term Q solution for increase in initial discharge rate Q_0. (f) The period between oscillation peaks drops to approx. 2 h.

(1)–(2) to obtain equation (3) for the mud propagation through an open conduit. The meaning of this equation becomes apparent after differentiation with time and combination with equation (5), yielding the physical implication of the discharge rate function

$$\frac{d^2Q}{dt^2} = \frac{A}{\rho h \beta V}\left(I - Q - S\frac{dr}{dt}\right) - \frac{8\pi\mu}{\rho A}\frac{dQ}{dt}. \tag{11}$$

Equation (11) is a second order differential equation with the form of damped, forced oscillators. This implies that the discharge rate Q has a natural tendency to oscillate with the period and damping controlled by the particular choice of parameters.

The general form of damped, forced harmonic oscillators is mostly given by

$$\ddot{x} + 2\gamma\dot{x} + \omega^2 x = F, \tag{12}$$

where x is the oscillator, $\dot{x}$ and $\ddot{x}$ its time derivatives, 2γ denotes the damping constant, ω is the characteristic, natural, angular frequency (giving the frequency of the oscillation via $f = \omega/(2\pi)$), and F is an external force (Iverson, 2008, and references therein). Compared to equation (11), we can extract some preliminary estimates on the effects caused by constant parameters. For instance, the root of the term $A/(\rho h \beta V)$ can be associated with the angular frequency of an oscillator. Therefore, we expect the conduit geometry, the mud compressibility, and the mud density to affect the frequency of the discharge rate solution Q. The factor $8\pi\mu/(\rho A)$ in equation (11) correspond to 2γ in equation (12). That means, that viscosity, density and conduit cross-section modifications are responsible for the damping of the oscillation for the mud volcano model. Finally, the expression $A(I-S(dr/dt))/(\rho h \beta V)$ is the external force acting on the oscillator equation (11), controlling the amplitude and the forced oscillation frequency (generally different from ω) of the solution. Here again, the conduit geometry and mud density occur, but the mud influx rate and the reservoir deflation become relevant for the discharge rate solution. This shows that by applying arbitrary equations for the influx rate I or the reservoir deflation dV/dt, respectively, the model can be extended to mimic external factors controlling the system behavior. For example, seismicity triggered pore-pressure or permeability changes (Miller et al., 1996; Miller and Nur, 2000) influence the hydrological properties of the system (Rojstaczer et al., 1995), i.e. flow path or flow velocity changes, resulting in an eruption recharge or seepage reinforcement (Manga and Brodsky, 2006).

This formulation gives the possibility to constrain parameter values in order to ensure oscillation of the system. The harmonic, damped system oscillates, if $\gamma^2 < \omega^2$ (underdamping), i.e., parameter combinations must be chosen carefully (overdamped and critically damped systems $\gamma^2 \geq \omega^2$ are not covered in this work).

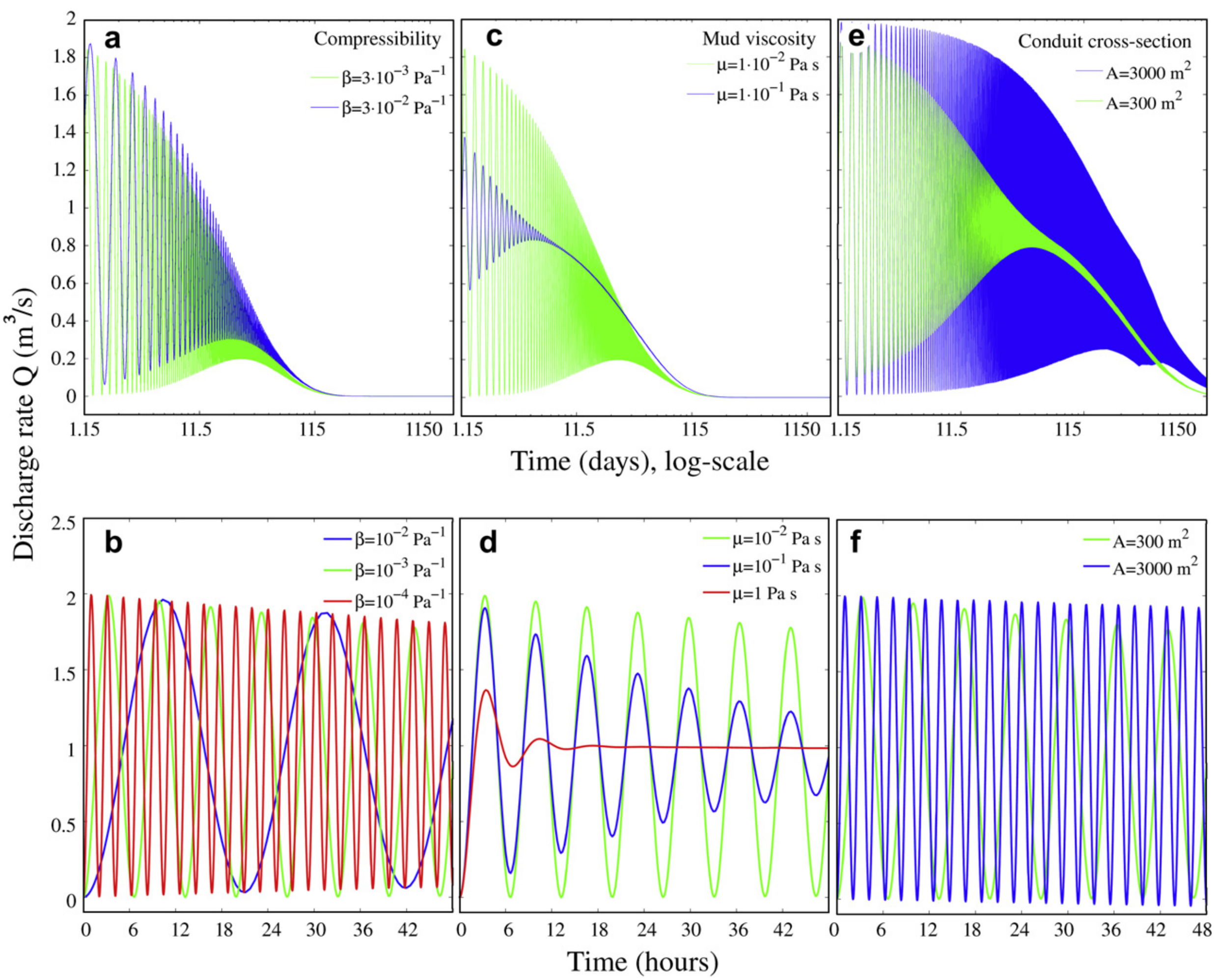

Fig. 4. Discharge rate Q for changes in parameter values. (a) Long-term behavior of mud compressibility β, decay occurs already after six months. (b) The smaller β, the shorter the discharge rate period. (c) Long-term discharge rate Q for increase in mud viscosity μ. Eruption period remains the same, oscillation amplitude decreases. (d) Reaching $\mu = 1$ Pa s, constant mud extrusion is observed following one significant discharge peak, decaying later on towards zero (not visible here). (e) Long-term discharge rate Q evolution for increase in A and (f) zoomed detail. Same effects can be seen for decrease in height h.

Furthermore, the model proposed here requires a minimum initial influx rate, as part of the external force F acting on the system. We do not include an explicit discussion of this here, because values of 0.1–2 m^3/s provide satisfactory results and are more than 4 orders of magnitude larger than the minimum influx rate.

4. Numerical results

We solve the system equation (3) using the boundary conditions (5) and (8) with a fourth order Runge-Kutta solver for ordinary differential equations with constant I_{const} or the solution to equation (10).

Parameter values listed in Table 1 are taken from the literature concerning mud volcanoes (Aslan et al., 2001; Bonini, 2007; Hovland et al., 1997), mud mobilization and eruption processes (Deville et al., 2006; Dimitrov, 2002; Mazzini et al., 2007), and drilling research (Wojtanowicz et al., 2001).

4.1. Mud flow with deflating reservoir

For the deflating reservoir case, we investigate the influence of physical parameters on the discharge rate Q, i.e. dynamic viscosity μ, conduit cross-sectional area A or the initial reservoir volume V_0. Figs. 3 and 4 show representative solutions to the discharge rate function Q. The influx rate I (initialized by an external triggering mechanism) causes an increase in reservoir pressure p and the beginning of mud discharge Q towards the surface. At the same time, the reservoir volume and its surface deflate, effecting a decrease in I. Consequently, Q decays over time in an oscillatory behavior with a constant period, reaching its equilibrium $Q = 0$ after certain characteristic times.

Further insights on the model behavior are provided by close-up views of the period of discharge rate oscillations depending on parameters and initial conditions. First, we discuss the changes on discharge rate behavior due to changes in the initial conditions shown in Fig. 3. Decreasing initial reservoir volume V_0 by one order of magnitude leads to a faster decay (from 3 years down to 6 months) of Q towards zero. This result is reasonable as a larger reservoir needs more time to deflate. The discharge rate period, however, is not affected (Fig. 3(b)).

Tests on the initial mud influx rate into the reservoir and discharge rate into the conduit are relevant for ongoing mud flows. Observations in Indonesia for example show that an ongoing mud extrusion can be disturbed, stimulated or damped by outside factors (Mazzini et al., 2007). Changes in the initial influx rate I_0 are plotted in Fig. 3(c) and (d). The extrusion period remains the same, but the amplitude decreases for smaller initial influx rates. The decay of mud discharge occurs at a later time. For an initial influx of 2 m^3/s, the maximum discharge rate Q is 4 m^3/s and reaches zero two months earlier than for $I_0 = 1$ m^3/s, taken for most calculations. In general, we solve the system of equations with $Q_0 = 0$ m^3/s. The consequences on the solution for an increase are shown in Fig. 3(e) for long-term behavior and in Fig. 3(f) as a close-up view. Discharge decays at the same time, but the oscillation amplitude and period change radically.

In the following, variations in parameter values are presented. A change in mud compressibility β or density ρ (not shown here) changes what is expected from damped oscillator equations. The decrease affects shorter periods (Fig. 4(b)), but the decay time remains the same (Fig. 4(a)). This result is intuitive; the lower the mud compressibility, the less mass per unit volume has to be extruded, ending up in shorter periods.

A comparison of plots in Fig. 4(c) and (d), shows the effect of increasing viscosity μ. The period remains the same (approximately 6 h between discharge rate peaks), but the stronger damping effect leads to a smaller oscillation amplitude. As the reservoir volume V and influx rate I are the same, the discharge rate reaches zero after six months like in the case shown in Fig. 4(a).

Changes in the conduit geometry produces similar results to the solution with different mud characteristics. Either an increase in cross-sectional area A or a decrease in conduit height h, respectively, shortens the period between discharge rate peaks, while maintaining the decay time. Fig. 4(e) and (f) shows the result for changing A.

Results from this model provide many physically meaningful scenarios for comparisons with observations of mud eruption processes (and geothermal geyser behavior (Ingebritsen and Rojstaczer, 1993)) as related to different parameters and initial conditions. However, a possible extension of this model from observations is to allow flow through fractured media, ending up in a full opening of the conduit (Gisler, 2009). This is an important aspect and will be the subject to future investigations.

4.2. Comparison with non-deflating reservoir

We conclude with solutions for a constant reservoir volume V (thus the reduced form of the boundary condition (8)), and a constant material influx rate I_{const}. In addition to the discharge rate solution plots, we show the consequences on the amount of

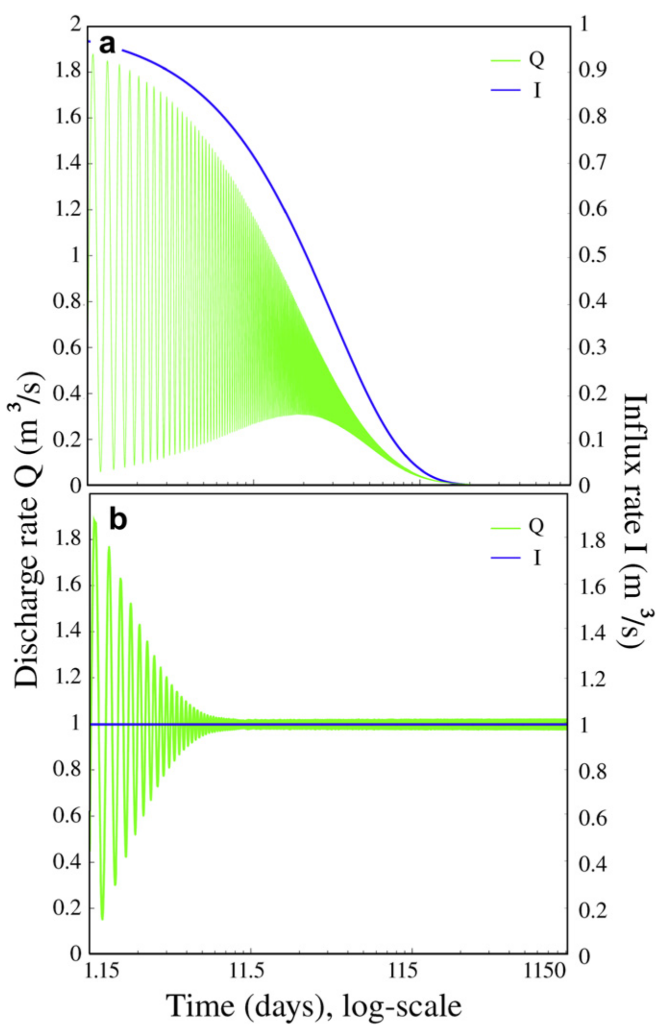

Fig. 5. (a) Solutions for the influx I and discharge Q functions calculated from equations (3), (5) and (10) given in previous paragraphs. As the mud reservoir deflates, I decreases, and Q decays towards zero following the solution graph of I. (b) Long-term discharge Q and influx I_{const} rates behavior. Q decays towards the value of I_{const}. Logarithmic time scale.

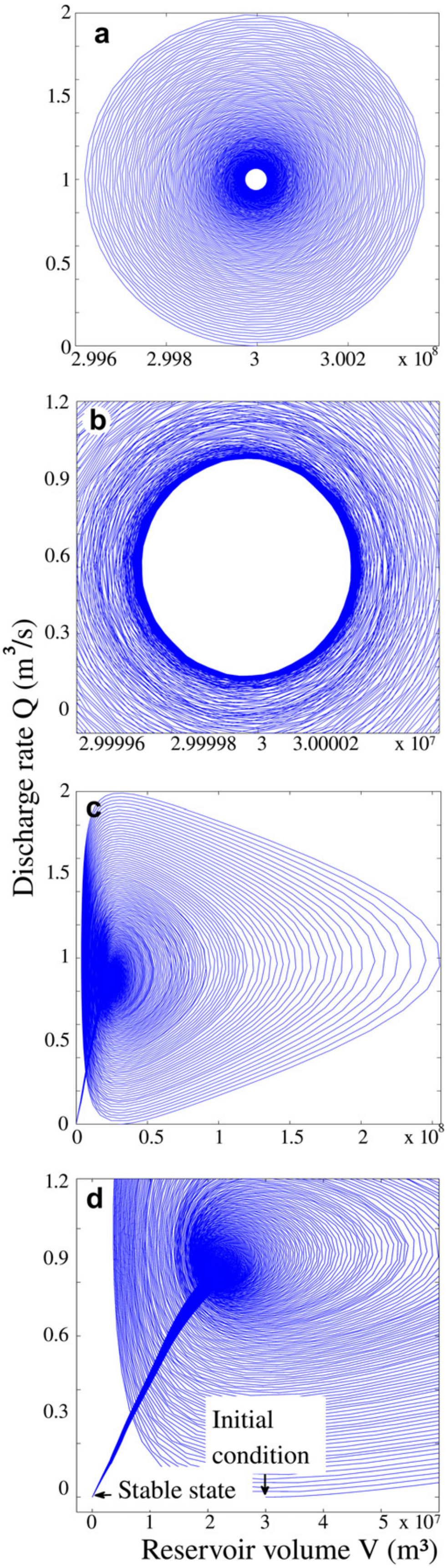

Fig. 6. (a) Limit cycle and (b) detailed view for constant influx rate. Stable state of Q is reached, when oscillating about the value of $I_{const} = 1$ m^3/s. Plotted versus fluctuating reservoir volume V. (c) Limit cycle, discharge rate Q vs. reservoir volume V, for an open mud conduit with deflating source. (d) Zoomed detail of figure c, showing up the initialization and equilibrium points. Periodic behavior with stable state in (0,0).

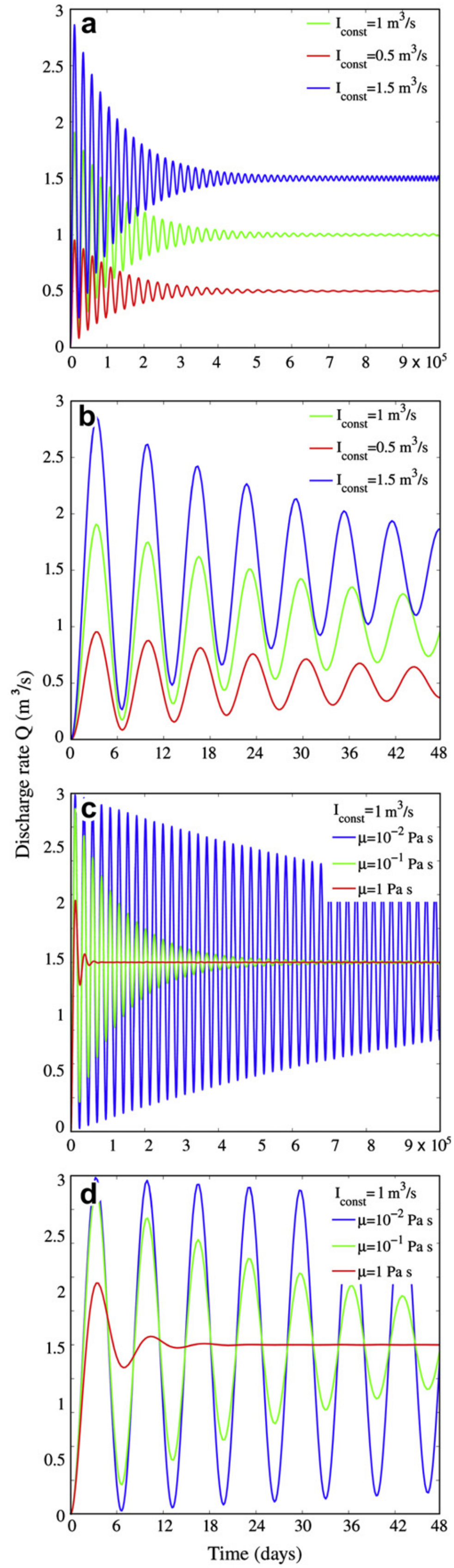

Fig. 7. Discharge rate Q functions for different values of I_{const}, (a) Long-term behavior and (b) zoomed detail. (c) Discharge rate Q for constant I and additional changes in μ. (d) As expected, the discharge rate oscillation amplitude changes.

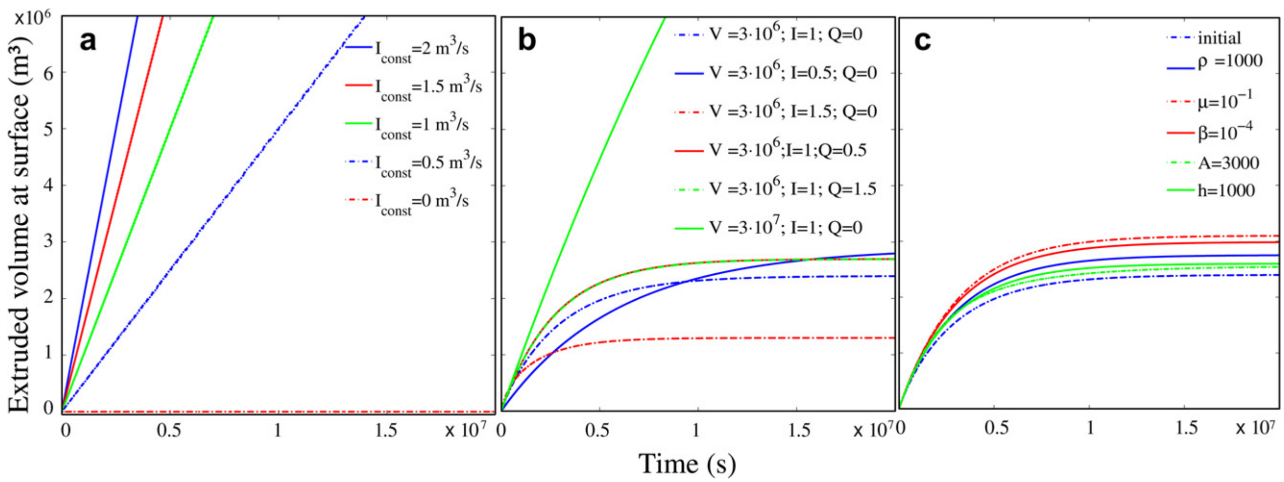

Fig. 8. Extruded volume at surface of the mud volcano. (a) Dependence on a constant influx I_{const}. (b) Dependence on initial conditions, when I is not constant. (c) Dependence on parameter choices, when I and Q decay to zero.

volume extruded at the surface due to the differences in the influx rate function.

Fig. 5(a) shows the influx rate function I and resulting discharge rate Q for an open conduit combined with a deflating reservoir, and Fig. 5(b) shows Q as result of a constant I. While in the first case the discharge rate decays towards zero, here the discharge rate decays to an oscillation about a constant equilibrium.

This is confirmed by the limit cycles of both processes (Fig. 6), where the discharge rate Q is plotted versus the reservoir volume V. For the non-deflating case, the discharge rate decays to oscillate about a constant equilibrium volume (Fig. 6(a) and (b)), while the cycle concentrates and reaches its final stable state at zero discharge rate Q and zero reservoir volume V, when considering deflating reservoirs (Fig. 6(c) and (d)).

Variations in I_{const} result in differences in final discharge rate values (see Fig. 7(a) and (b)), and additional parameter modifications affect changes in oscillation period and/or amplitude as seen previously. The graphs for changes in viscosity μ are plotted as representative examples in Fig. 7(c) and (d).

Plots for the remaining parameter changes are not included. However, they yield similar results in period and amplitude changes as presented for the case of a deflating reservoir, with the difference that the discharge rate Q decays towards, and oscillates around I_{const}.

Finally, we examine the differences on the extruded mud volume at the surface caused by variations in model assumptions. Fig. 8(a) shows the estimated mud volume at the surface after 3 years of significant activity. Due to the constant influx I_{const} and non-changing source conditions, the extruded volume increases linearly with time.

For the case where the reservoir deflates and consequently the mud source is finite, we expect a saturation of the extruded mud volume. Indeed, this effect can be seen in Fig. 8(b) and (c), where different constant mud volumes are reached depending on changes in initial conditions and parameters.

5. Discussion and conclusions

We have developed a simple model to describe periodic mud volcano extrusion processes. One conduit geometry and two mud reservoir estimates were investigated. Our results demonstrate the dependence of the discharge rate and extruded mud volume solutions on the initial conditions and parameter choices. We interpret the oscillation period of the discharge rate as time between eruptions or maximum seepage, and the oscillation amplitude as the eruption or seepage intensity. In addition, we presented solutions for the estimated mud volume extruded at the surface.

Using a deflating mud source reservoir, the initial reservoir volume was identified as the main influence on the decay time of the discharge process. The initial influx rate also affects the decay of the process, but the differences are less significant. However, the amplitude is strongly driven by the influx rate, which we presume to be initialized by a triggering event that we do not attempt to identify. Other oscillation amplitude driving forces are the initial discharge rate, interesting for cases of disturbance of ongoing fluid flows, and the mud viscosity. Other parameters such as the conduit cross-sectional area, height and mud compressibility mainly influence the oscillation period and therefore the time between eruption peaks.

Investigations of a non-deflating reservoir (e.g. persistent supply and influx rates) show similar dependence of the initial conditions and parameters, but revealed an important modification possibility for seepage settings lasting for decades, as a continous influx of material produces a pulsating discharge rate around a non-zero equilibrium.

Neglecting the initial triggering and conduit opening process, we examined mud ascent through an open mud volcano conduit. For application to real mud volcanoes, a combination of this approach with fractured rock models is more reasonable, that is, a fractured system at depth ending up in an open conduit in shallow regions close to the surface. This will be addressed in future studies.

Direct comparison with observations is difficult because of the lack of parameter constraints such as density, viscosity and discharge rates. The discharge rates of the LUSI mud volcano are reasonably well-constrained, but other parameters are not. For certain parameter combinations the oscillator equation is overdamped, i.e. the discharge rate function loses its periodic characteristic. Therefore our models do not only cover cyclic mud volcano behavior, but can also be used for single eruption investigations (not included here). However, cyclic behavior is very common, and our model can be adjusted to different mud volcano settings.

This simple model captures the basic dynamics of an eruption or seepage process, particularly the controls on the periodicity and decay time. Recognizing that initial values such as the influx rate function I and reservoir volume estimates, and that parameters influence the system behavior, this model can mimic a wide range of mud volcano eruption situations. Possible extensions of this model would be to include fully compressible flow, the use of a non-constant viscosity, conduit erosion, and introducing various (possibly arbitrary) influx rate rules. We suggest that this model forms the basis for comparisons with data on mud volcano discharge rates and mud extrusion to the surface.

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