Longevity Estimate of the Sidoarjo Mud Eruption 'Lusi'

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When will the eruption end?

- We need to understand how the mud source functions.
- We develop a model of the mud source and plumbing system.
- Certain unknown parameters affect the model outcome. We take the Monte Carlo approach to explore the sensitivity of the outcome to each of these parameters.
- We make statistical predictions of longevity.

Key Assumptions

- The mud comes from the upper Kalibeng, between 1200-1800 meters depth
  - Microfossils, Temperature, Clay mineralogy (Mazzini et al.2007), Geodesy (Fukushima et al. 2009)
- Water comes from upper Kalibeng after initial stage
  - Consistent with 30% porosity (Tanikawa et al. 2010). If Davies et al. (2011) value of 10-13% is more realistic, a deep aquifer is needed to explain 30% water content at surface (Bayuaji et al. 2009). An additional water source may have played a role during the initial stage of the eruption.
- Mud exhibits a yielding rheology
  - Laboratory rheology experiments (Manga et al. 2009, Rudolph and Manga 2010), Field observations (Kopf et al. 2009), also inability of relief wells to penetrate mud source (Sawolo personal communication).
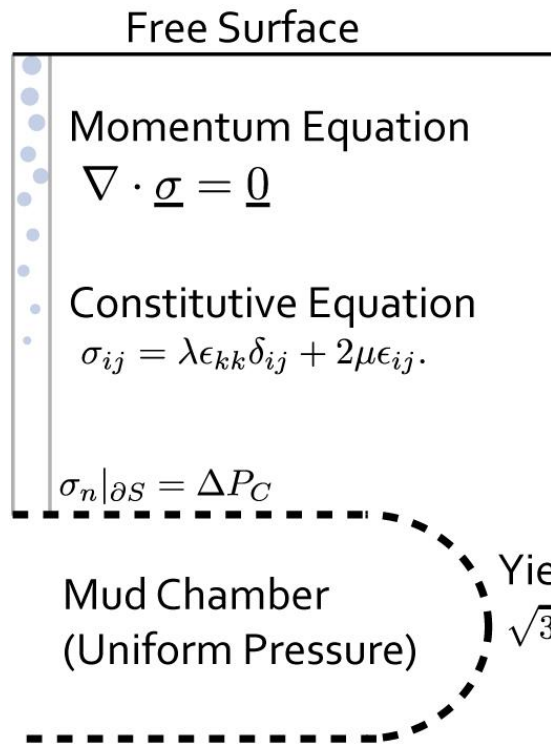
Model - Schematic
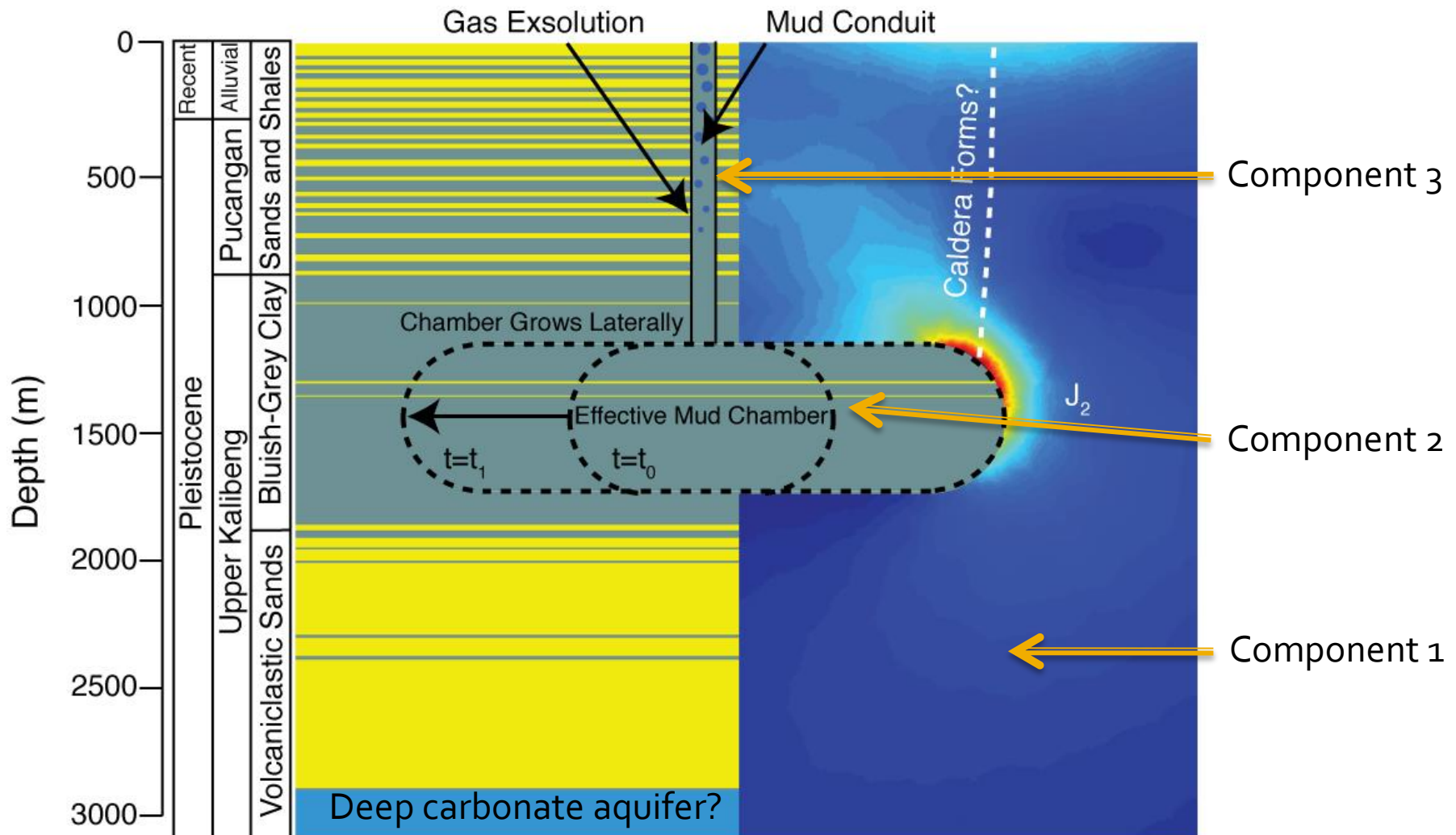
Stratigraphy from Mazzini et al. 2007

J2 – Second deviatoric stress invariant

Model – Mud Reservoir

Free Surface

Momentum Equation

$$\nabla \cdot \underline{\sigma} = \underline{0}$$

Constitutive Equation

$$\sigma_{ij} = \lambda \epsilon_{kk} \delta_{ij} + 2\mu \epsilon_{ij}.$$

$$\sigma_n|_{\partial S} = \Delta P_C$$

Mud Chamber
(Uniform Pressure)

Yielding Criterion:

$$\sqrt{3J_2} > \sigma_{y,chamber}$$

Surroundings are modeled as a linear elastic medium **Yielding occurs** according to von Mises criterion. Chamber pressure tends to decrease as material erupts, but low hydraulic diffusivity of surroundings keep pore pressures high. When surroundings yield and become incorporated into chamber, chamber pressure increases.

The von Mises stress is a scalar measure of deviatoric stresses
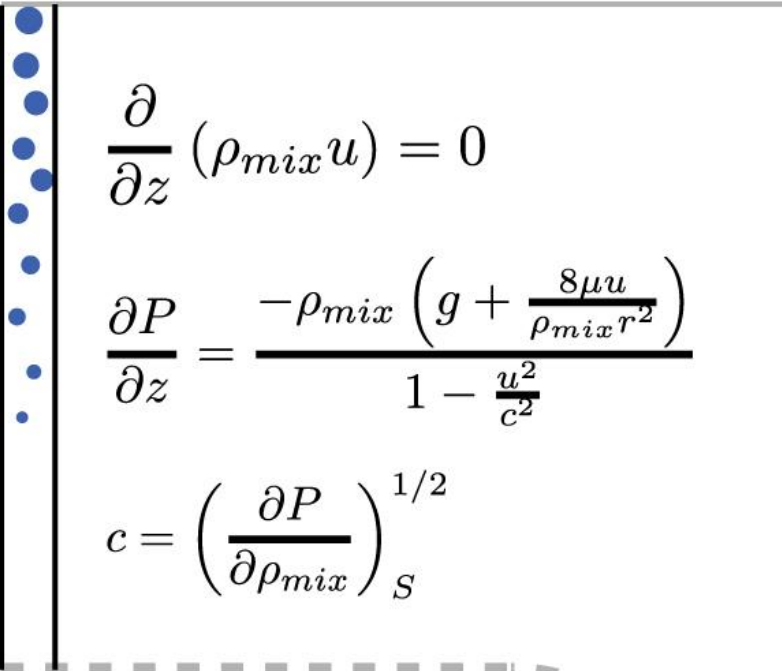
Pressure-Volume relation:

$$\rho_0 V_{0,C} + \int_0^t \dot{M}(\tau) d\tau = V_C(P)/v_S(P)$$

P   chamber pressure,
$V_C(P)$ current deformed chamber volume
$v_s(P)$ specific volume of (particles+liquid+gas)

Initial mass of system – (Cumulative mass removed) =Mass of material remaining in chamber

Model – Mud Conduit

$$P = P_{atm}$$

$$\frac{\partial}{\partial z}\left(\rho_{mix}u\right) = 0$$

$$\frac{\partial P}{\partial z} = \frac{-\rho_{mix}\left(g + \frac{8\mu u}{\rho_{mix}r^2}\right)}{1 - \frac{u^2}{c^2}}$$

$$c = \left(\frac{\partial P}{\partial \rho_{mix}}\right)_S^{1/2}$$

$$P = P_{chamber}$$

We model the conduit in 1D

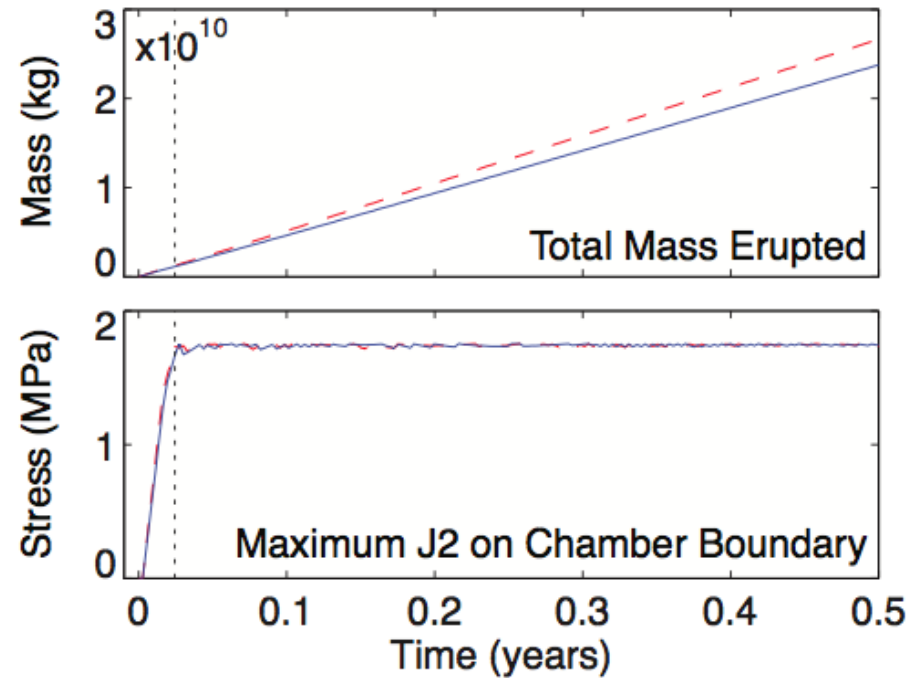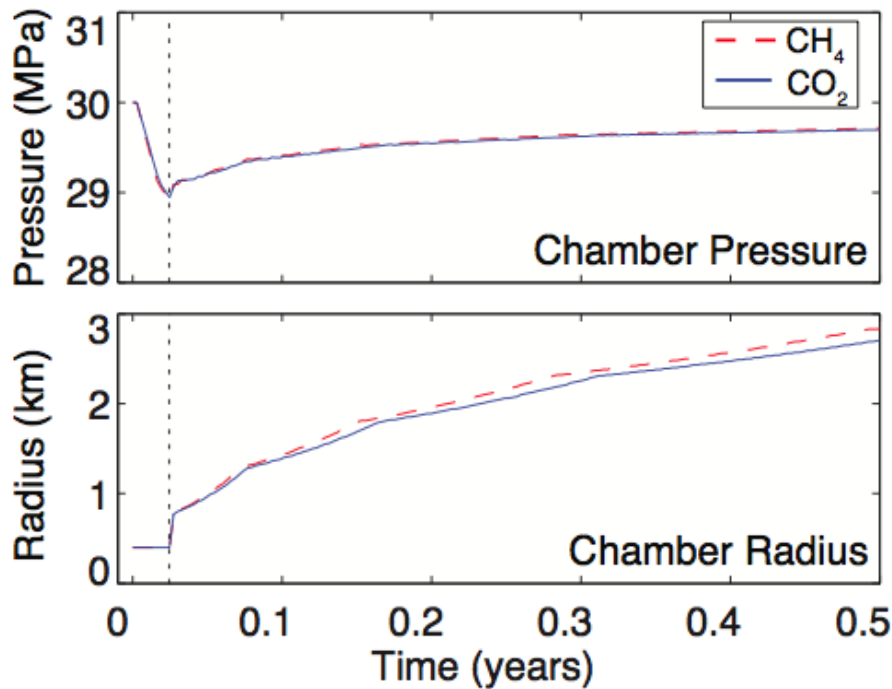Atmospheric pressure at top boundary

Chamber pressure at bottom boundary

Conservation of mass and momentum

Sound speed computed from equation of state

We use the CO_2-CH_4-H_2O equation of state of Duan et al. 1992a,b.

Results – Example

Key observations:
1) Yielding begins at dashed line
2) After yielding, chamber begins to expand
3) After yielding, stresses near chamber are held constant at the yield stress
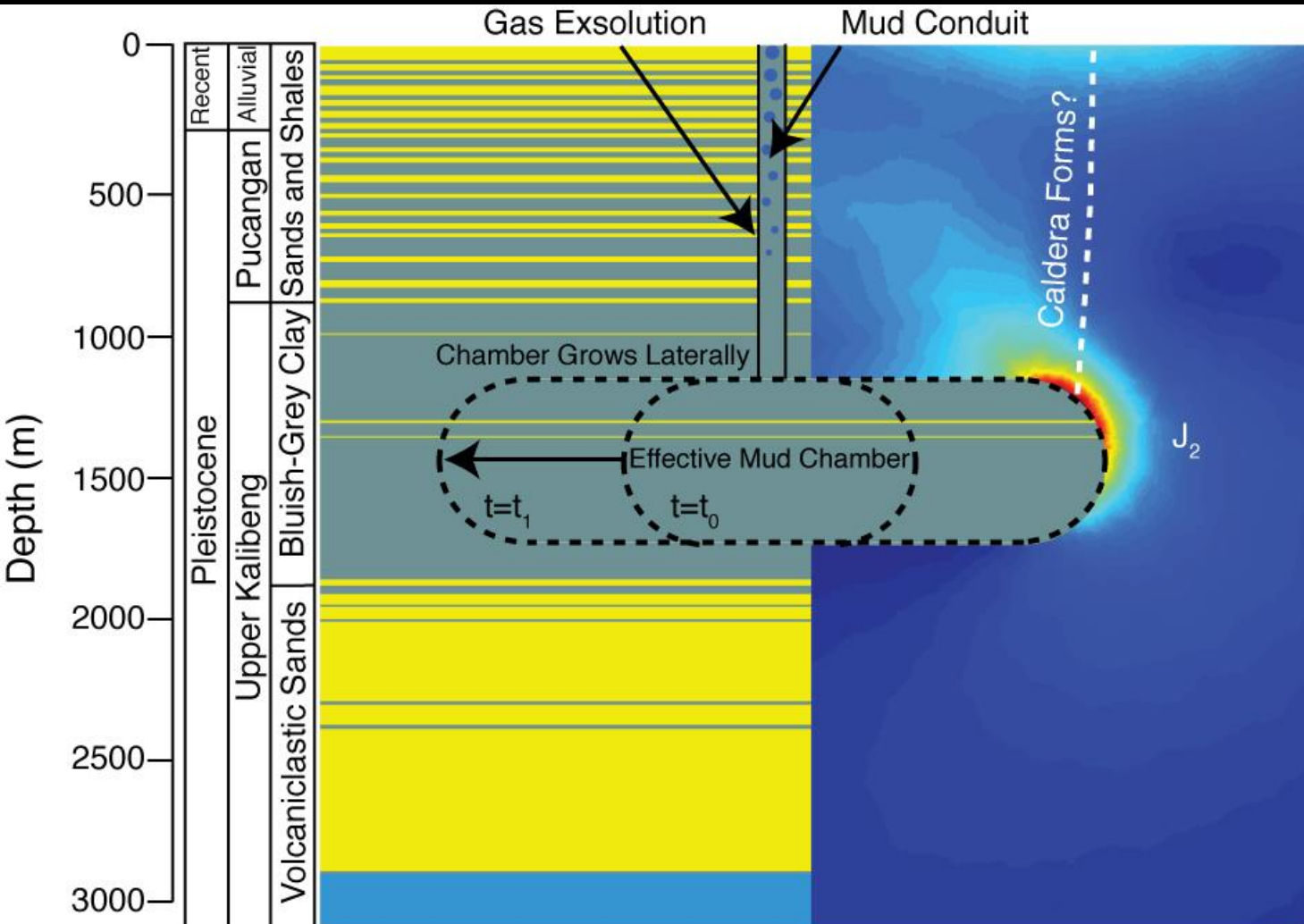4) Discharge is very nearly constant

Model – Eruption Termination

Two possibilities:

1) Chamber pressure becomes too small.
2) A caldera forms.

Eruption may not truly end when caldera forms, but our model is no longer applicable. We assume that a caldera forms when J_2 exceeds a critical value in the region between the tip of the chamber and the land surface.

Unknowns

Monte Carlo Simulations

Parameter	Constant or Variable?	Range (1 standard deviation) or value
E (Young's modulus)	Variable	10^7-10^9 Pa
ν (Poisson's ratio)	Variable	.05-.25
Mud Yield Strength	Variable	10^5-10^7 Pa
Yield Strength for Caldera	Variable	10^6-10^8 Pa
Water Content (by Volume)	Constant	30 %
Mud Viscosity	Constant*	10^4 Pa-s
Conduit Radius	Constant*	1.4 m (CH_4) or 8.5 m (CO_2)
Gas Concentration	Constant	1% CO_2 or 0.5% CH_4

*For a given gas composition and viscosity, we iteratively find a radius that satisfies observational constraints

Results – Sensitivity to Parameters
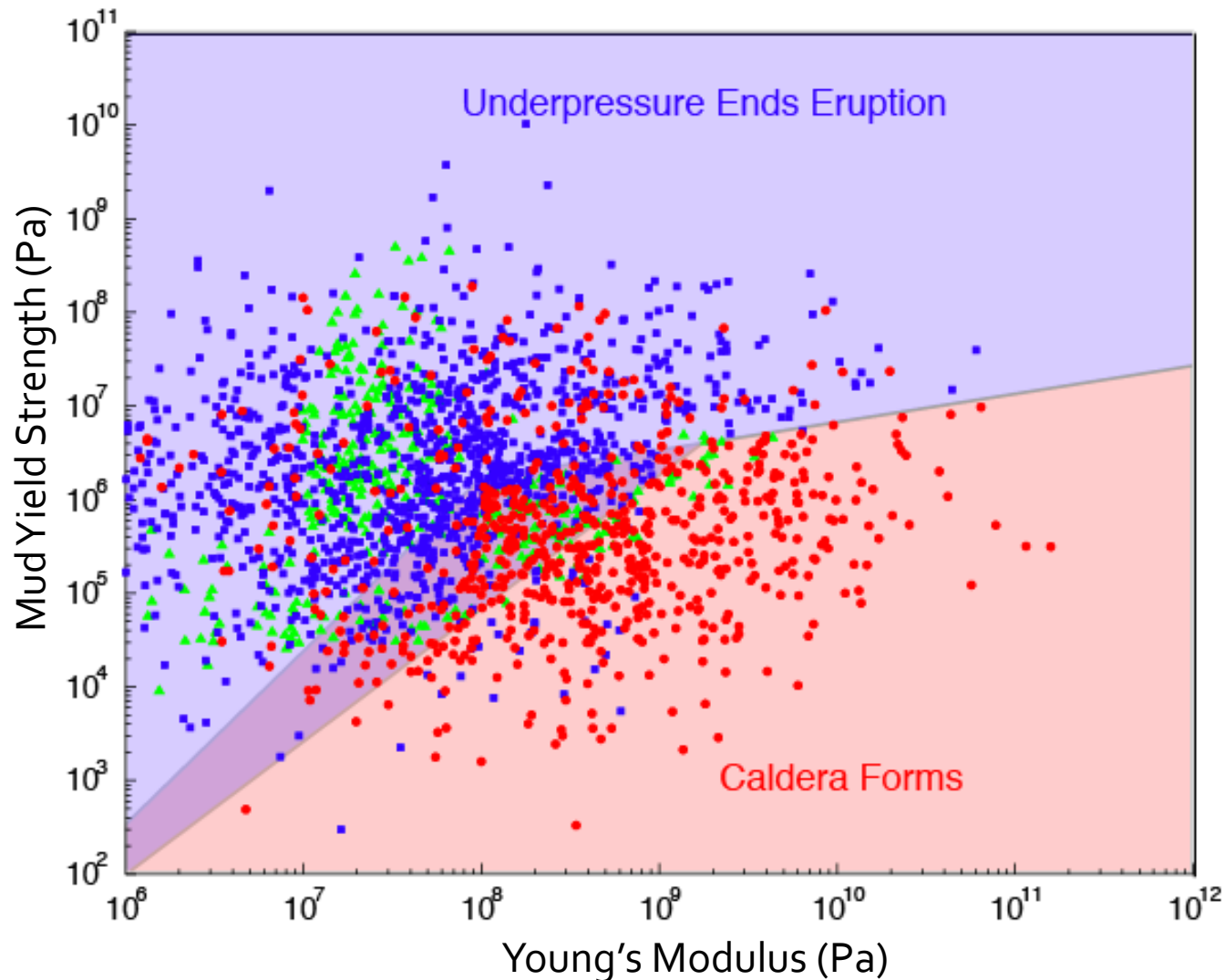
Chamber yield stress, E are most important

Poisson's ratio unimportant

Results – Longevity Predictions

Three Scenarios:
1) Gaussian variable distributions
2) Uniform variable distributions within 1 standard deviation
3) Uniform variable distributions within 2 standard deviations

Scenario:	1 - Gaussian	2 – Flat (1σ)	3 – Flat (2σ)
33%	21	27	14
50%	40	50	25
66%	84	>100	52

We assume here that the eruption lasts more than 5 years.

Note: Our predictions are somewhat longer than those of Davies et al. 2011

Results - Outcome
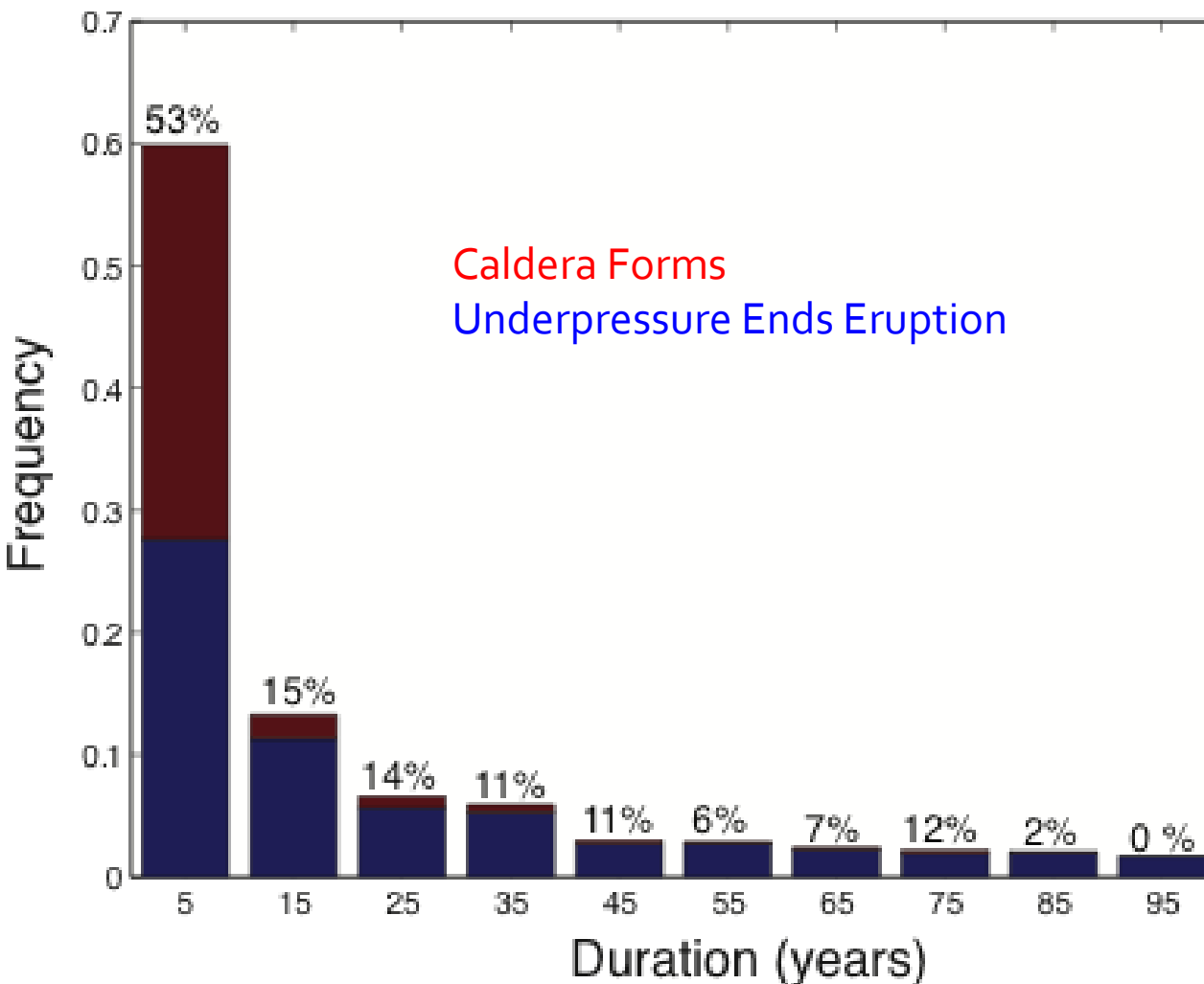
Longer eruptions are less likely to produce a caldera.

Caldera Forms
Underpressure Ends Eruption

Complete results in manuscript to appear in Earth and Planetary Science Letters. Available at: http://seismo.berkeley.edu/~max

How can we improve predictions?

- Geochemical constraints on:
  - 1) Source of water and relative contributions from deep and shallow sources.
  - 2) How the relative contributions have changed over time. We can perform analyses, but need samples.
- Better estimates of discharge as a function of time.
- Constraints on mud source rheology (yield strength). We can measure this, but need samples.
- Constraints on mud source geometry.